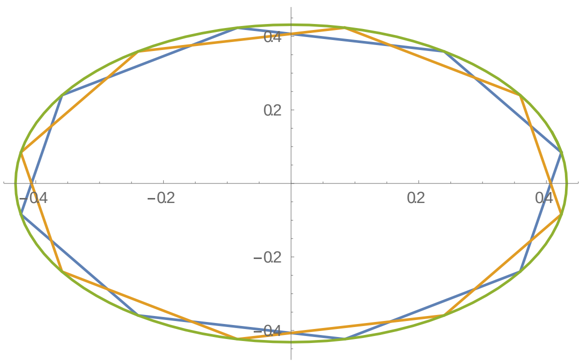
The Perrin Sequence, Octagons, Ellipses and Phasors

In the last chapter I described the complex octagon of sequences which applies to all polynomials which have an associated sequence and a real algebraic integer. Polynomials have roots which may contain both real and complex and/or imaginary roots. In this chapter the concept of the complex octagon is applied to complex roots. Higher order polynomials can have multiple real and complex roots. For complex roots, both the complex and its conjugate are equivalent to a single root since they have the same modulus. The number of real roots and individual moduli is equal to the number of complex octagons associated with the polynomial.

The 8 vertices of the complex octagon and its conjugates lie on an ellipse which is an oval curve intersecting these vertices in the complex plane. In the last chapter the polar plots showed regular octagons with an outer circular radius given by the squared modulus I defined as CR2. When these points are plotted in the complex plane the vertices of the octagons lie on an outer radius of an ellipse.



Plot on the complex plane of the real solution to the polynomial $x^3 - 2x^2 - 1 = 0$ with discriminant = -59. The complex R value are calculated from equations [8], [9] and [11] in Chapter 52 from a new De= 36.514.. has a modulus = 0.43175..

The interesting fact of the ellipse is that all points on the ellipse have the **same** modulus even though the complex R value varies around the curve. The R values are described by both the modulus and the argument such that R(ϕ) = Abs(R)*exp(i*ϕ) where Abs(R) is the modulus and ϕ is an angle in radians. There exist an infinite number of complex R values calculated from rotation of R, some that can be roots of different polynomials. This chapter describes equations to find these polynomials. Note that since the modulus squared is used to calculate the root of the cubic polynomial this ellipse is specific to the real root of the cubic polynomial of discriminant De = 36.514.. In this case $x = (2/\mathrm{Abs}[R]^2)^{(1/3)}$ = 2.2055694304005 … the same as calculated in Chapter 20, Table II. In Chapter 28 the g Weber invariant is calculated in radical form for D = -59 from the q octic continued fraction. There is a simple modular equation which can be used to compare the roots x, one of the g invariants from its integer discriminant D and another to a general discriminant such as De calculated from the k modulus.

[1] $x = \mathrm{Exp}[\pi\sqrt{D}/24] * \prod_{k=0}^{200}(1 + \mathrm{Exp}[-(2k+1)\pi\sqrt{D}])$

where D can be any discriminant (positive D or De) and the upper limit to the product is a large number to improve precision of x. The plus sign used for odd or non-integer discriminants can be replaced with a minus sign for even discriminant D.

The subject of this book started with the examination of the Perrin sequence. It is here appropriate to understand the structure of the complex and the associated ellipses using the polynomial, plastic number, and complex roots of $\mathbf{x^3 - x - 1}$ as an example. This example can be used as a model for any order polynomial. Unlike previous chapters describing calculations which result in sequence properties such as polynomial roots, and sequence number, the ellipses of a polynomial are obtained from the known roots to provide information on modular structure. Elliptic functions were described in Chapter 45 to provide trajectories and orbits of objects under gravitational influence. These orbits are elliptical, and the trajectories are not always closed. Polynomial orbits are closed, and a single polynomial can have m + n/2 associated ellipses where m is the number of real roots and n the number of complex and conjugate pairs. The sum m + n equals the number of roots or the order Ord of the polynomial.

Each ellipse R(ϕ) has multiple complex values that are roots of other polynomials and in most cases these polynomials are of higher order than the original polynomial. However, these higher order polynomials contain a subgroup of roots that are of modulus Abs[R(ϕ)], the remaining roots having other moduli. Roots that are of modulus Abs[R(ϕ)] can be complex, real, or imaginary. The common factor in all the polynomials of R(ϕ) is the discriminant. The discriminant of higher associated polynomials contains after factoring, the original polynomial discriminant.

In the simple example of the Perrin sequence, we begin with the discriminant D = -23. The real root, the plastic number and the conjugate pair of roots were calculated in the first Chapter and several other chapters of this book. I will label the roots as φ and φc and φc* for the complex root and its conjugate.

The CR2 or $\text{Abs}[R]^2$ values can be easily found,

[2a] $CR2r = 2/(\varphi)^3$

[2b] $CR2c = (\varphi c)(\varphi c^*)$

where r refers to the real solution and c the complex. Note that there is a difference in defining CR2c since the solution is already complex whereas the complex R value for the real root is to be calculated. This complex can be found using equations [8], [9] and [11] from Chapter 52. The associated De = 4.57529.. which when substituted for D in equation [1] gives the plastic number. Note that using D = 23 results in $\varphi * \sqrt{2}$ in agreement with Weber's g invariant.

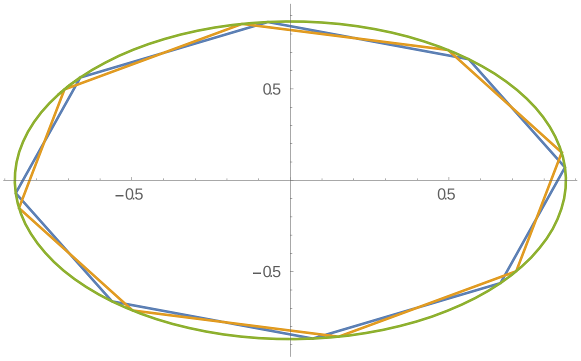
The calculated value Rr(ϕ) is also found to agree with equation [10], Ch 52.

[3a] $\text{Abs}[Rr(\phi)^8 - 1] = 1$

A similar calculation can be done with CR2c to find Rc(ϕ) such that,

[3b] $\text{Abs}[Rc(\phi)^8 - 1] = 1$

I find that Abs[Rc(ϕ)] = Abs[φc] = Abs[φc*] but the arguments do not agree since these R values all lie on the same ellipse but are rotated by different degrees around the complex plane. A comparison of the two octagons on the ellipse shows that Rc(ϕ)] is rotated slightly more than φc about the plane.



Plot on the complex plane of the complex solution φc (blue) to the polynomial x^3 -x-1 = 0 with discriminant = -23. The complex Rc value (red) calculated from equations [8], [9] and [11] in Chapter 52 from a new De= 6.140.. Both octagons have an equal modulus = 0.8688.. and lie on the same ellipse.

The arguments of Rr, Rc and are φc found to be 0.16166, 0.17596, and 2.4377 radians, respectively. Rr has a modulus = 0.9275.. and this defines the root from equation [2a] above. The complex value of Rc does not match a solution to **x^3 -x-1 = 0.** *Mathematica* has a unique function called **RootApproximant** that can locate the polynomial associated with the root up to the precision of the root. For Rc the root is associated to the polynomial $1 - 3z^8 + 13z^{16} - 21z^{24} + 15z^{32} - 5z^{40} + z^{48}$. There are 48 roots to this equation. Sixteen roots have a modulus = 0.8688.., the remaining have different moduli. The discriminant factors as {{2,144},{7,32},{23,24}} showing it contains the prime 23 as found in the original polynomial.

It would be convenient to find an equation for the ellipse which includes the root φc. We would also like the angle of the first vertex of the octagon or close to 11.25 degrees. Rotation of the argument of φc by 225 degrees aligns the vertex with the argument of Rr. The exact number is,

[4] $\phi = -0.080126993474200618126439790 \ldots + \mathrm{Arg}[\mathrm{Rr}]$

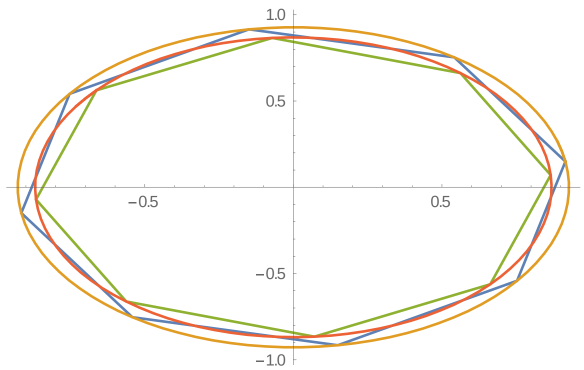
showing that rotation puts the angle within 0.080.. radians of the angle for Rr.

The equations for the complex and real ellipses are the magnitude times exponential of the argument times the imaginary number I,

[5] $(-1)^{j/4} * \mathrm{Abs}[\mathrm{Rc}] * \mathrm{Exp}[I * \phi]$

[6] $(-1)^{j/4} * \mathrm{Abs}[\mathrm{Rr}] * \mathrm{Exp}[I * Arg[Rr]]$

The number $(-1)^{j/4}$ rotates the vertices in the complex plane with j ranging from 0 to 8. The plot of these vertices produces two octagons at different radius and tilts about the (x,y) plane.



Plot on the complex plane of the complex solution (inner ellipse) to the polynomial x^3 -x-1 = 0 with discriminant = -23 and the Rr associated with the plastic number (outer ellipse). Each octagon has different modulus and arguments as described by equations [5] and [6].

Starting with the inner ellipse it is of interest to start rotating equation [5] by multiplying the angle ϕ by integers and fractions. In the table below results of multiplication of the angle ϕ by f and setting j to various values is shown. Note *Mathematica* returns the root approximate with a polynomial in a variable #1. **Table I**

f	j	Root Approximant	Discriminant
1	3	$-1 - \#1 + \#1^3$	{−1,1},{23,1}
1	2	$1 - 3\#1^4 + 2\#1^8 + \#1^{12}$	{2,24},{23,4}
2	3	$1 - \#1^4 + 9\#1^8 + 35\#1^{12} + 53\#1^{16} + 3\#1^{20} + \#1^{24}$	{2,48},{3,24},{5,8},{7,8},{23,12},{59,8}
3	2	$1 + 13\#1^4 + 62\#1^8 - 137\#1^{12} + 292\#1^{16} - 32\#1^{20} + \#1^{24}$	{2,120},{5,8},{7,24},{11,8},{23,12}
4	3	$1 + 33\#1^4 + 518\#1^8 + 535\#1^{12} + 1484\#1^{16} - 36\#1^{20} + \#1^{24}$	{2,96},{5,8},{7,24},{19,8},{23,12},{59,8},{173,8}
5	3	$1 + 5\#1 + 15\#1^2 + 21\#1^3 + 13\#1^4 + 3\#1^5 + \#1^6$	{−1,1},{7,4},{23,3}
5/2	3/2	$1 + 35\#1^8 - 94\#1^{12} + 128\#1^{16} - 12\#1^{20} + \#1^{24}$	{2,72},{19,8},{23,12},{6607,8}
5/4	3/4	$1 + 5\#1^8 - 10\#1^{16} - 11\#1^{24} + 208\#1^{32} + 22\#1^{40} + \#1^{48}$	{2,192},{3,48},{7,32},{19,16},{23,24}

Integer and fractional multiplication of the argument as described by equations [5] to calculate new complex roots and locate their associated polynomials on the complex ellipse. In these examples j indicates rotation by integer or fractional increments is also required. Note that in all discriminants powers of 23 occur.

In a similar method as above, rotation of equation [6] is achieved by multiplying the argument of Rr by a positive or negative integer as shown in the **Table II** below.

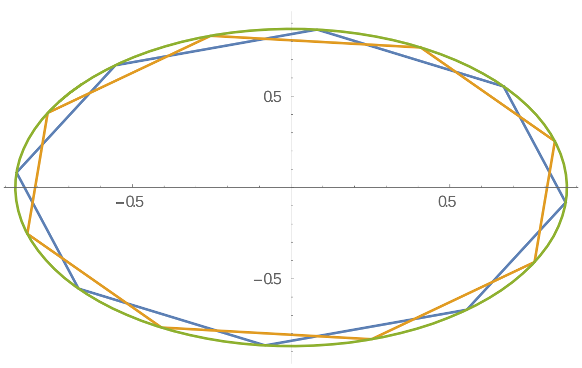
f	j	Root Approximant	Discriminant
2	0	$4096 + 32768\#1^4 + 335104\#1^8 - 197632\#1^{12} + 647072\#1^{16} - 1600\#1^{20} + \#1^{24}$	{2,348},{5,16},{7,4},{11,4},{23,8},{199,4},{211,4},{709,8},{393947,8}
4	0	$64 - 3584\#1^2 + 49232\#1^4 + 28032\#1^6 + 69624\#1^8 + 224\#1^{10} + \#1^{12}$	{2,90},{3,12},{7,2},{11,2},{23,4},{199,2},{211,2},{307,4},{427591,4}
8	2	$64 + 188224\#1^2 + 384467152\#1^4 - 760176752\#1^6 + 520013864\#1^8 - 45064\#1^{10} + \#1^{12}$	{2,138},{3,12},{7,2},{11,2},{23,4},{199,2},{211,2},{138603787850391193163,4}
-8	2	$64 + 188224\#1^2 + 384467152\#1^4 - 760176752\#1^6 + 520013864\#1^8 - 45064\#1^{10} + \#1^{12}$	{2,138},{3,12},{7,2},{11,2},{23,4},{199,2},{211,2},{138603787850391193163,4}
-4	2	$64 - 3584\#1^2 + 49232\#1^4 + 28032\#1^6 + 69624\#1^8 + 224\#1^{10} + \#1^{12}$	{2,90},{3,12},{7,2},{11,2},{23,4},{199,2},{211,2},{307,4},{427591,4}
-2	6	$4096 + 32768\#1^4 + 335104\#1^8 - 197632\#1^{12} + 647072\#1^{16} - 1600\#1^{20} + \#1^{24}$	{2,348},{5,16},{7,4},{11,4},{23,8},{199,4},{211,4},{709,8},{393947,8}

Note that the same root approximants are obtained for both positive and negative f values. Also, a rotation about the octagon by even increments of 90 degrees also results in the same root. These results are accurate to 90 decimal places for the real and imaginary part of the complex number.

It is more difficult to find polynomials of roots to equation [6] for integral f values than for equation [5]. Another method for finding f is to seek not integer of f using the **FindRoot** function in *Mathematica*. This requires using one of the results in the above tables and finding another f value that is non-integral. For example, using the complex value from the first table for f =2, j= 3 a new f1 = 11.632007.. is found resulting in the same root approximant. Choosing another higher value found at f2 = 319.85625.. also results in the same root approximant. These numbers are then multiplied by integers and added or subtracted resulting in the following table of polynomial root approximants. **Table III**

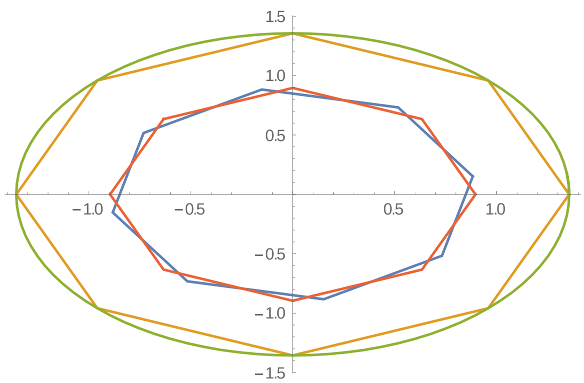
f	j	Root Approximant	Discriminant
f1	0	$1 - \#1^4 + 9\#1^8 + 35\#1^{12} + 53\#1^{16} + 3\#1^{20} + \#1^{24}$	{2,48},{3,24},{5,8},{7,8},{23,12},{59,8}
f2	3	$1 - \#1^2 + \#1^4 - 7\#1^6 + 11\#1^8 - 5\#1^{10} + \#1^{12}$	{2,12},{3,12},{23,6}
f2-f1	3	$1 + 2\#1^4 + \#1^8 + \#1^{12}$	{2,24},{23,4}
3f2+2f1	1	$1 + 13\#1^2 + 3480\#1^4 - 441\#1^6 + 6094\#1^8 + 156\#1^{10} + \#1^{12}$	{2,24},{19,4},{23,6},{43,4},{101,4},{2540789,4}
3f2-6f1	5	$1 + 6\#1^2 + 127\#1^4 + 168\#1^6 + 186\#1^8 - 26\#1^{10} + \#1^{12}$	{2,24},{5,4},{11,4},{23,6},{101,4},{317,4}
6f1-3f2	5	$1 - 6\#1^2 + 127\#1^4 - 168\#1^6 + 186\#1^8 + 26\#1^{10} + \#1^{12}$	{2,24},{5,4},{11,4},{23,6},{101,4},{317,4}

Many more examples can be shown. The two examples at the bottom of the table illustrate the symmetry about the complex octagon with equation [5] calculating a rotated 90-degree conjugate value and producing the same discriminant.



Plot on the complex plane of the complex solution (blue) to a polynomial with f= 5/4, J = 3/4 with argument of 61.9 degrees (last entry in Table 1) compared to the polynomial solution to f = 6f1-3f2, j = 5 with argument 39.5 degrees in Table III. Compare to the complex solution φc of argument 4.7 degrees from the real axis. Each octagon has equal modulus and arguments as described by equations [5].

Equations [5] and [6] are known as "phasors" or phase vectors in physics and engineering. They simplify the solution of current and power in alternating voltage circuits containing various components. Usually, a second frequency or phase is added to the argument which adds a time component. This time component sweeps around the ellipse and creates a sin wave of frequency (f+w) and of amplitude (Abs[R]). In circuits each component is a phasor which adds and subtracts and multiplies to create a phasor whose real part represents current or power. In a similar way phasors representing roots to polynomials can be added and multiplied. In the examples above Rc is multiplied by the last complex root in Table I, squared, and added to φc. The result is rotated by -90 degrees and the real part results in the algebraic integer 1.124559.. which is a solution to $-23 + 9z^2 + 6z^4 + z^6 = 0$. The discriminant of this equation factors to {2,6}, {3,12}, {23,3}. All magnitudes of these complex numbers are equal but adding phasors of different magnitude is common in circuit design. By analogy, sequence and polynomial design is possible such as developing orthogonal polynomials in the ellipse of other orthogonal polynomials. Orthogonal polynomials have real roots and can be plotted on the real axis. The phasor for an orthogonal polynomial would only represent the real roots. The symmetry of these phasors developed for the roots of a Hermite polynomial are found to be all real or mixed real and complex roots.



Plot on the complex plane of a rotated real solution (outer octagon) to the hermite polynomial $15x - 10x^3 + x^5$. The inner octagons correspond to phasor Rr values from one real solution. One rotated solution (blue) falls off the real axis and produces a co-orthogonal polynomial $1/16 * (3375x - 2200x^3 + 16x^5)$.

Due to their symmetry, phasor solutions as above, show a co-orthogonality on integration with Hermite polynomials of similar order with expected values of $\sqrt{2\pi} * n!$ but are not self-orthogonal. Since orthogonal polynomials and phasors are of importance in signal processing, neural networks, quantum harmonics, probability, and combinatorics further research on applying phasor ellipses and octagons to polynomials can lead to new insights in these fields of study.

Finally, hybrid phasors can be found which find polynomials of a mixed prime discriminant. In Table I we found a 12th order polynomial containing 23 in the discriminant for f* $\phi = \phi$ and Abs[Rc]= 0.86883 calculated from Perrin's polynomial. A corresponding calculation of these parameters for the polynomial **x^3 -2 x^2-1** of discriminant (-59) gives $\phi 59 = 0.1532463..$ ($\phi 23$ = 0.08154..) and Abs[Rc59] = 0.673348 … Hybrids prepared from $(-1)^{1/2} * \text{Abs[Rc]} * \text{Exp}[I * \phi 59]$ and $(-1)^{1/2} * \text{Abs[Rc59]} * \text{Exp}[I * \phi 23]$ yield an 18th order polynomial with discriminants of {primes, power} containing {23,6}, {59,9} and {23,9}, {59,8}, respectively. Hybid phasors can be found for any combination of primes by mixing rotation of the phases on one ellipse with the magnitude of another ellipse!

The Perrin Sequence, Octagons, Ellipses and Phasors- Appendix

(All terms are described in Chapter 53)

The Perrin Sequence as phasors: D = -23 Polynomial $x^3 - x - 1$

Rr=$0.91543924769273599430965771875432864193634295179132579708898726755554745266480163010067030\!9 + 0.14929970457991879327420644245771580274513346506901938348409864830517433493179896355118461\!4i$

Rc= $0.85542052552421509557894020785551819308243020520995547979359876811722664129382684738427584\!2 + 0.15209664939954604479466856932487374435942552276948788597489707208936653332471212785700614\!8i$

$\phi = -0.0801269934742006181264397907484312940021708686621032044925759039964122451147988167512953\!25 + \mathrm{Arg}[\mathrm{Rr}]$

P(m) =

$$\mathrm{Abs}[2(\mathrm{Rr}^{-2}\mathrm{Exp}[2i\mathrm{Arg}[\mathrm{Rr}]])^{1/3}]^m + ((-1)^{3/4} * \mathrm{Abs}[\mathrm{Rc}] * \mathrm{Exp}[i * (\phi)])^m + ((-1)^{5/4} * \mathrm{Abs}[\mathrm{Rc}] * \mathrm{Exp}[i * (-\phi)])^m$$

Other Sequences as phasors: D = -59 Polynomial $x^3 - 2x^2 - 1$

Rr=$0.4234619124235201122543493429494147689114950146906572655799852266835184 + 0.0841985906117193402887221809143080507949194720755899386625058036964879\!9i$

Rc= $0.6607545722060450538011288937460496642395378435617375114738561505776557 + 0.129618852121171636798039295364061288904787360116428892047201211664762\!3i$

$\phi = 0.15324633498049940963833672870492381762877799819360406400465223127396\!66$

P59(m) =

$$\mathrm{Abs}[2(\mathrm{Rr}^{-2}\mathrm{Exp}[2i\mathrm{Arg}[\mathrm{Rr}]])^{1/3}]^m + ((-1)^{2/4} * \mathrm{Abs}[\mathrm{Rc}] * \mathrm{Exp}[i * (\phi)])^m + ((-1)^{6/4} * \mathrm{Abs}[\mathrm{Rc}] * \mathrm{Exp}[i * (-\phi)])^m$$

Other Sequences as phasors: D = -17 Polynomial $x^3 + 3x^2 - 3x + 2$

Rr=$-0.18005140584993027938028034277485502401910666342837760595291682018457\!75 - 0.0358144362969512783601165278050181297330053246573356927563124953377000\!2i$

Rc= $0.7029004954016755361482783316691619679264586834642583950479566304875783 + 0.136660968028211085426985117803192947612697224113027058106885307826170\!6i$

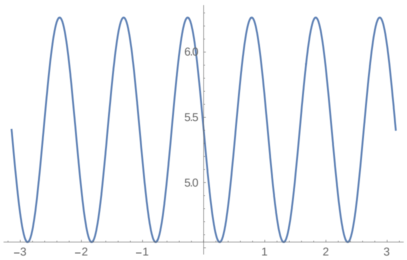
$\phi = -3.0362500069897631492215194167016005507249478156211797616436692939074868$

P17(m) =

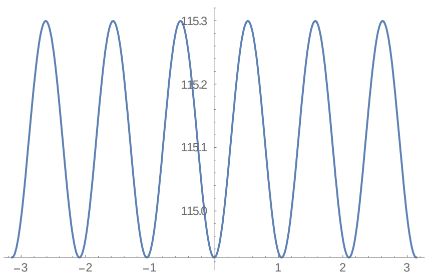
$$(-1)^m\mathrm{Abs}[2(\mathrm{Rr}^{-2}\mathrm{Exp}[2i\mathrm{Arg}[\mathrm{Rr}]])^{1/3}]^m + ((-1)^{5/4} * \mathrm{Abs}[\mathrm{Rc}] * \mathrm{Exp}[i * (\phi)])^m + ((-1)^{3/4} * \mathrm{Abs}[\mathrm{Rc}] * \mathrm{Exp}[i * (-\phi)])^m$$

Graphs showing magnitude of phasor for m = 6 and as ϕ varies from $-\pi$ to $+\pi$.

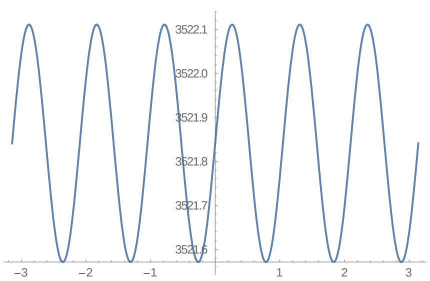
Perrin Sequence D= -23:



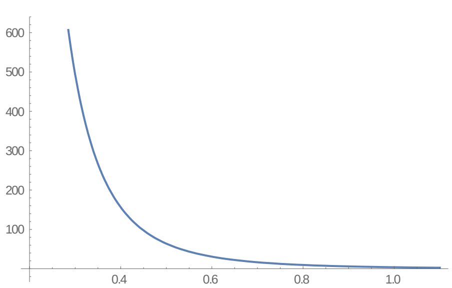
D = -59



D= -17



The values of the 6th Sequence element for a **mixed** phasor with Rc at D=-59 and x = Abs[Rr] varied from 0.1 to 1.1. The value at x = Abs[Rr] for D = -59 is 115. All other integers are possible sequence values at m = 6 for this mixed phasor as a function of x.



A Fourier and Inverse Fourier Transform of Sequences

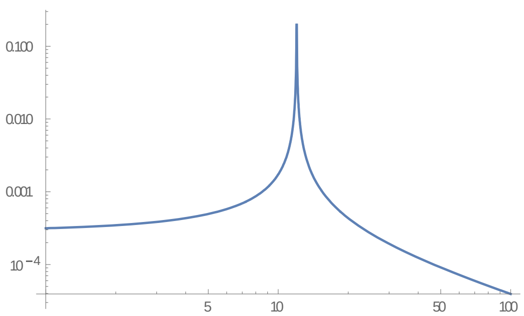
Each solution of the roots of a polynomial equation can be expressed as a phasor showing that these roots are periodic in the complex plane. The integer sequence from the polynomial is obtained from the sum of powers of these roots, and as shown above, is a periodic function of the argument of the phasor. This periodicity is expressed as a sin wave composed of the sum of component waves of different magnitude and frequency. Furthermore, each power of the sequence results in a different sine wave of varied magnitude and frequency.

A Fourier transform can be applied to the cubic functions P(m), P(17) and P(59) above. This transform has an interesting ability to change the function P(m, ϕ) into a function of frequency, P(m, w). As I will demonstrate below this transform has many advantages in finding the numerical sequence of any cubic polynomial and potentially of polynomials of any degree.

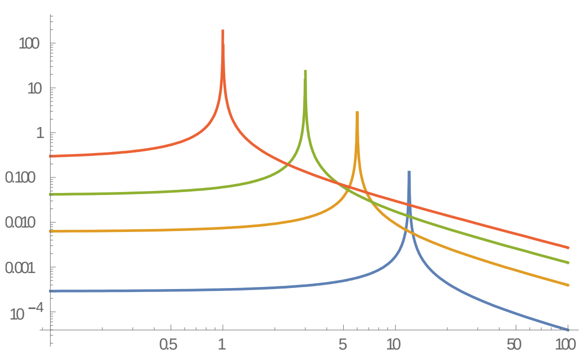
Mathematica is used to perform the Fourier transform on P(59) at m = 12. (R59 = Rr, FR59 = Rc above)

$$\text{FourierTransform}\left[\text{Abs}\left[\left(2\,R59^{-2}\,e^{-2 i \text{Arg}[R59]}\right)^{1/3}\right]^{12} + \left((-1)^{2/4} * \text{Abs}[FR59] * \text{Exp}[i * (t)]\right)^{12} + \left((-1)^{6/4} * \text{Abs}[FR59] * \text{Exp}[i * (-t)]\right)^{12} \text{UnitStep}[t], t, \omega\right]$$

The result in a log/log or Bode plot shows a peak at m = 12 on the abscissa,



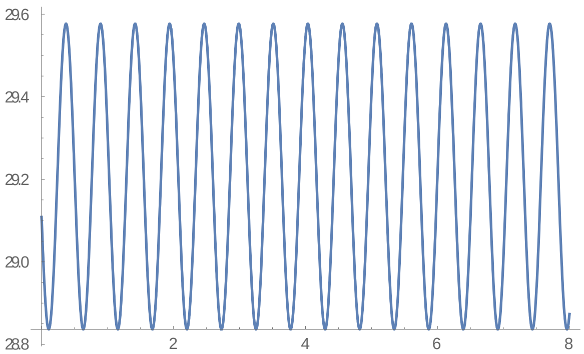
Graphs at various m values show similar curves with the spike at the given m value. The meaning of the ordinate is ambiguous since the scale of the abscissa determines the height of the ordinate and does not appear to represent any useful information. Plot showing transform graphs for m = 1, 3, 6 and 12;



Next, I remove the values of the variables and create dummy variables Rc and Rr and change the argument to (t + ϕ59) performing the transform for m = 12 with the following output.

$$\frac{ie^{-12i\phi 59}\text{Abs}[\text{Rc}]^{12}}{\sqrt{2\pi}(-12+\omega)}+e^{-12i\phi 59}\sqrt{\frac{\pi}{2}}\text{Abs}[\text{Rc}]^{12}\text{DiracDelta}[-12+\omega]+\frac{16\sqrt{2\pi}\text{DiracDelta}[\omega]}{\text{Abs}[\text{Rr}]^{8}}+e^{12i\phi 59}\sqrt{2\pi}\text{Abs}[\text{Rc}]^{12}\text{DiracDelta}[12+\omega]$$

Applying the Inverse Fourier transform to this expression plugging in the above values for R59, Rc59 and $\phi 59$, plotting I get back the sine wave.



Note that the result expresses the actual sequence value of 29 at periodic values of the sine wave. However, another advantage is found by taking the Fourier transform followed by its inverse: **the exponential terms, $e^{12i\phi 59}$, above can be removed without changing the result**!

There is also a pattern in the expression for various values of m as well as factors at each of the 4 terms. Examining a pattern which occurs after every 4 values of m the general Fourier transform can be found as follows,

$$\frac{(-i)^{\text{Mod}[(m-1),4]}*(C\text{Rc})^{m}}{\sqrt{2\pi}(-m+\omega)}+(-i)^{\text{Mod}[m,4]}*\sqrt{\frac{\pi}{2}}(C\text{Rc})^{m}\text{DiracDelta}[-m+\omega]+\text{sgn}*\frac{2^{((2m+3)/6)}*\sqrt{\pi}\text{DiracDelta}[\omega]}{(C\text{Rr})^{2m/3}}+(i)^{\text{Mod}[(m),4]}*\sqrt{2\pi}(C\text{Rc})^{m}\text{DiracDelta}[m+\omega]$$

The expression for P(m, w) is only a function of m, CRc, and CRr where CRc and CRr are the squares of the magnitudes of the complex numbers Rc and Rr. This equation is found to apply to any cubic polynomial and will provide the complete sequence using only the real numbers CRc and CRr! The value of sgn is either, +/- 1 depending on the sign of the real root.

The sequence table for the first 20 terms for D =-59 above is calculated from the Inverse Fourier Transform in *Matthematica*.

Input: $\text{Table}[\text{Round}[\text{Evaluate@InverseFourierTransform}[\frac{(-i)^{\text{Mod}[(m-1),4]}*(\text{CRc})^{m}}{\sqrt{2\pi}(-m+\omega)}+(-i)^{\text{Mod}[m,4]}*\sqrt{\frac{\pi}{2}}(\text{CRc})^{m}\text{DiracDelta}[-m+\omega]+\text{sgn}*\frac{2^{((2m+3)/6)}*\sqrt{\pi}\text{DiracDelta}[\omega]}{(\text{CRr})^{2m/3}}+(i)^{\text{Mod}[(m),4]}*\sqrt{2\pi}(\text{CRc})^{m}\text{DiracDelta}[m+\omega],\omega,0]],\{m,1,17\}]$

Output: {2,4,11,24,52,115,254,560,1235,2724,6008,13251,29226,64460,142171,313568,691596}

Where CRr = $CR2r^{1/2}$ and CRc = $CR2c^{1/2}$ from equations {2a] and [2b] above. Note that CRc and CRr must be less than about 0.9 to calculate accurately the sequence.

Bell Polynomials, Hypergeometric functions, ISPs and Inverse Fourier Transforms express integer sequences.