Chalkboard 1: PERRIN EQUATION [Origin]

[On-line Encyclopedia of Integer Sequences] OEIS A001608

The sequence is generated from roots of a solution to the Elliptic equation:

$g(z) = z^3 + \alpha * z + \beta$

SOLVE for 3 ROOTS 1 real (algebraic number and 2 complex numbers)

$\alpha := -1$
$\beta := -1$

1. Numerical solution [MathCad]

Solve for roots of function g(z):

$g(z) := z^3 - z - 1$

Provide an approximate value for the solution, and modify until the solver is converging appropriately. It is helpful to graph the function to find a value that is reasonably close to the root as a starting guess.

$z := 0.1 + 0i$

Real root:

$r13 := \mathrm{root}(g(z), z)$
$r13 = 1.325$

Note: For a complex solution, input a complex guess value.

$z := 0.5 + 2i$

$r23 := \mathrm{root}(g(z), z)$
$r23 = -0.662 + 0.562i$

$z := 0.5 - 2i$

$r33 := \mathrm{root}(g(z), z)$

$r33 = -0.662 - 0.562i$

2. Solution based on Kra and Simanca Notices of AMS 59,(3) pp372 (2012)

   Solve for two numbers a and b given by:

$$b := \left[\frac{-\beta - \left(\beta^2 + \frac{4\cdot\alpha^3}{27}\right)^{\frac{1}{2}}}{2}\right]^{\frac{1}{3}}$$

$$a := \left[\frac{-\beta + \left(\beta^2 + \frac{4\cdot\alpha^3}{27}\right)^{\frac{1}{2}}}{2}\right]^{\frac{1}{3}}$$

Define:

$$r1 := a + b$$

$$r2 := a\cdot\exp\left(2\frac{\pi\cdot i}{3}\right) + b\cdot\exp\left(4\frac{\pi\cdot i}{3}\right)$$

$$\varepsilon := \exp\left(\frac{2\cdot\pi\cdot i}{3}\right)$$

$$r3 := a\cdot\exp\left(4\cdot\frac{\pi\cdot i}{3}\right) + b\cdot\exp\left(2\cdot\frac{\pi\cdot i}{3}\right)$$

SOLUTIONS

$a + b = 1.325$ [Kra] {only given to 3 decimals}

$\boxed{r13 = 1.32471796}$ (numerical solution to 8 decimals)

$$a\cdot\varepsilon + b\cdot\varepsilon^2 = -0.662 + 0.562i$$

$$\boxed{r23 = -0.662 + 0.562i}$$

$$a\cdot\varepsilon^2 + b\cdot\varepsilon = -0.662 - 0.562i$$

$$\boxed{r33 = -0.662 - 0.562i}$$

Some properties of a and b:

$$3\cdot a\cdot b = 1$$

$$a^3 + b^3 = 1$$

Check solutions in the function g:

$g(r1) = 0$

$g(r2) = 0$

$g(r3) = 0$

The Perrin Sequence S1 is generated from these roots of the elliptic equation above: Example 6th power S1(6)

$n := 6$

$S1 := r13^n + r23^n + r33^n$

$S1 = 5$

Note that S1 is a series of integers

3,0,2,3,2,5,**5**,7,10……

Also the Inverse Perrin number (-n) results in another sequence SN1. Example of 6^{th} power SN1(6)

Application of this sequence will be discussed in another Chalkboard

$SN1 := r13^{-n} + r23^{-n} + r33^{-n}$

$SN1 = -2$

Note SN1 is a series of positive and negative integers:

3,-1,1,2,-3,4,**-2**,-1,5….

OEIS A078712 [Also known as sequence of negative n]

Infinite Sequences:

For n an integer equal to or greater than zero:

SN(n+3) = SN(n) + SN(n+1)

SN1(n+3) = SN1(n)-SN(n+2)

Next Chalkboard #2 will discuss finite sequences of the Perrin sequence.

RT